Altruism:
how others affect our contribution to public goods

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Abstract

Mainstream theoretical results on voluntary contributions to public goods are inconsistent with empirical data: while standard models predict that the dominant strategy is to free-ride by not contributing to the public account, experimental research usually detects positive contributions. Moreover, the contribution magnitude has been showed to be sensitive to many features of the environment: the very same decision maker may behave differently according to the setting in which the action takes place. To account for this evidence, I modify the standard *Voluntary Contribution Mechanism* by adding a non-monetary reward in the decision maker's payoff which factors in the warm-glow, that is the private emotional reward entailed by the contribution, the number of beneficiaries of the public good and the income inequality among the players. I obtain three main results. First, the decision maker's optimal choice can be to contribute with a part of her income to the public account. Second, the optimal choice is smoothly increasing in the Marginal Per Capita Return of the public good. Third, all else equal, the optimal choice responds positively to the number of beneficiaries of the public good and negatively to the relative income of the other players. This implies that both the quantitative and the qualitative compositions of the community affect the optimal contribution. To test this last result, I develop three experimental designs: two Dictator Games adjusted for public goods aimed at exploring the relation between the voluntary contribution to a public good and the number of its beneficiaries, and a one-shot Voluntary Contribution Mechanism with asymmetric rewards to analyse the relation between the voluntary contribution and the income inequality among the players.

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Introduction

Theoretical predictions on voluntary contributions to public goods are not consistent with empirical evidence: the most common model used to formalize the choice between a private good and a public one — the linear *Voluntary Contribution Mechanism* — shows that, as long as her marginal cost of contribution exceeds her private marginal benefit, the decision maker's optimal strategy is always to free-ride by not contributing to the public account.

Despite this clear suggestion from theoretical analysis, results from experimental research usually show that people contribute to the public good with positive — sometimes even consistent — shares of their endowments, ignoring their dominant strategy in the attempt to cooperate with other players. Moreover, the optimal amount of contribution has been shown to be sensitive to many features of the environment: the very same decision maker may behave differently depending on the type of setting in which she is required to act.

Relying on these observations, it appears worthy to develop a modified version of the *Voluntary Contribution Mechanism* that embeds some features of the environment. Among all, I choose the most related ones to the social dimension of public goods games, in order to focus on the relevance of the type of community that interacts with the decision maker.

The first feature I take into account is the private emotional reward entailed by contribution. This phenomenon has been discussed by many authors over time, like Olson (1965) and Becker (1974), but it has been finally formalized by Andreoni (1990), receiving the name of *warm-glow*. To get the intuition, look at this advertisement by the American Red Cross:[1]

"Feel good about yourself — Give blood!"

The slogan is based on the assumption that the decision maker is going to be satisfied by her contribution, regardless the total amount of blood the organization will be able to collect. In other words, the agent is characterized by *impure altruism*, since her payoff depends not only on the overall value of the public good, but also on the size of the private contribution.

The second feature is the number of beneficiaries of the public good: if the decision maker is pleased to help other people, then she will prefer to help $(N + 1)$ subjects rather than N, as long as the public good is pure. To ensure this property, the analysis is focused on a *VCM* without provision point, with a constant Marginal Per Capita Return with respect to the size of the community. The first feature means that there is not a minimum level of total contributions that must be reached in order to provide the public good, while the second one means that the private marginal gain from the public good does not depend on the group size.

Finally, income inequality among the players is taken into account. Previous research has already shown that it has a negative impact on the optimal contribution. However, the novelty

[1] The slogan is from [5].

of this study relies on the intuition that each decision maker does not consider the other players' income in absolute terms; instead, she compares it to her own wealth and — all else equal — she will prefer to help poorer agents. This idea is based on economic justifications, as well as on ethical ones.

These variables are introduced in the *Voluntary Contribution Mechanism* one by one by adding a non-monetary reward in the decision maker's payoff labelled *pleasure function*, which — in its final formulation $R_i(x_i, N, \mathbf{d_i})$ — denotes the pleasure felt by agent i after a contribution of amount x_i to a public good with N beneficiaries, whose income inequality with the decision maker is described by the vector $\mathbf{d_i}$.

This modified version of the *VCM* — called *altruistic VCM* — improves upon the theoretical benchmark since its main predictions better fit the experimental findings. In fact, while the standard *VCM* predicts that the optimal contribution is either zero or the entire endowment depending on the value of the Marginal Per Capita Return of the public good, in the *altruistic VCM* the decision maker's optimal choice can be to contribute just partially to the public account. Moreover, the optimal choice becomes smoothly increasing in the Marginal Per Capita Return of the public good, which accords with the empirical findings by Isaac and Walker (1984), but is not predicted by existing models. Furthermore, the new model suggests that — all else equal — the decision maker's optimal choice responds positively to the number of beneficiaries of the public good and negatively to the relative income of each other player. The main implication is that both the *quantitative* and *qualitative* compositions of the players' group matter: consequently, the very same decision maker will act differently depending on the group with which she is supposed to cooperate. In particular, some groups are *cursed* in advance by lack of cooperation.

The remainder of this essay is organized as follows. Section 1 reviews the main results obtained in related literature. Section 2 describes the standard Voluntary Contribution Mechanism. Section 3 analyses the *VCM* with a pleasure function depending on the size of contribution. Section 4 introduces the number of beneficiaries into the pleasure function. Section 5 takes income inequality among players into account. Section 6 describes three experimental designs aimed at testing some of the results of the model. Section 7 concludes.

1 Main literature

This sections aims at offering an overview of the main results by past research on voluntary provision of public goods.

One of the first economists who discussed the contrast between individual incentives and group interest was Lindahl in 1919. In 1954, Samuelson presented a theoretical formulation, while Ledyard and Roberts provided a proof in 1974. In the very same period, political scientists studied it as *tragedy of the commons* (Hardin, 1968) or a *problem of collective action* (Olson, 1971), while social psychologists called it *a social dilemma* (Dawes, 1980).

However, the methodical analysis of the phenomenon started with the support of experiments. In particular, three main research groups systematically exerted effort to understand the mechanisms that lead to cooperation in public goods games: Marwell in sociology, Dawes and Orbell in psychology and political science, Isaac and Walker in economics. The common finding of these studies was a strong deviation from the *free-rider* prediction, that researchers tried to explain by considering different features of the environment.

One of the first aspects they took into account was the timing of the game: all the experiments involving a repeated game found contributions to decline with repetition (see Isaac, McCue and Plott (1985), Andreoni (1988) and Croson (1996)). However, both Andreoni and Croson found a significant *"restart effect"*: subjects would increase their contribution if the game was played again from the top. This finding implied that positive contributions could not be explained in terms of lack of experience, since subjects showed a positive willingness to contribute even after having been trained. Actually, results on economics training are quite contradictory: Marwell and Ames (1981) got that contributions were significantly lower if and only if the subjects of their experiment were graduate students from economics. However, other studies, like the one by Isaac, McCue, and Plott (1985), refused this observation.

The explanation in terms of pure altruism is generally rejected by literature since it lacks predictive power: Warr (1982) proved theoretically that government grants should crowd out voluntary gifts dollar-for-dollar under the assumption of pure altruism. The result was then extended to subsidies by Bernheim (1986) and Andreoni (1988). For this reason, Andreoni (1990) developed a theoretical model where an *impure altruistic* agent gains a non-monetary reward in terms of satisfaction — called *warm-glow* — which is a function of the individual contribution rather than the overall value of the public good. Authors like Olson (1965) and Becker (1974) had already suggested the same intuition.

Similarly, Marwell and Ames (1979) tried to explain the voluntary contribution in terms of interest to the public good, finding a positive correlation. On this path, Isaac and Walker (1984) analyzed the relation between the voluntary contribution and the Marginal Per Capita Return (*MPCR*) of the public good: their results suggested a positive dependence even for $MPCR < 1$, that is a case where the optimal strategy suggested by theoretical models is always to free-ride.

Many authors analyzed the effect of inequality varying the endowments rather than the value of the *MPCR*: while Marwell and Ames (1979) reported that inequality in endowments had no effect on contributions, Bagnoli and McKee (1991), as well as Rapaport and Suleiman (1993), found a negative effect. Later, Anderson, Mellor and Milyo (2004) repeated the experiment first introduced by Marwell and Ames, and then replied by Isaac at al. (1984). However, they introduced inequality in the environment using additional fixed payments rather than modifying the endowments, in order to keep constant the feasible set of actions of the players. Their findings showed that inequality in the distribution of fixed payments reduced the subjects' contributions.

Results on group size are ambiguous, and they often depend on the mechanism design. Marwell and Ames (1979) tested the effect of the group size on contributions in a one-shot public good game with provision point: they showed that smaller groups gave in average higher amounts to the public pot; however, the differences were not significant at the 0.05 level. Later, Isaac and Walker (1988) run a similar experiment introducing repetition and removing the provision point. In this way, they reversed Marwell and Ames' results since they observed more free-riding in smaller groups, holding constant the *MPCR*. However, also their results were statistically weak. Kahneman and Knetsch (1992) got a similar finding: giving rose when the subjects were asked to contribute to a local public good first, then to a regional public good and finally to a national one.

Relying on the observation of the importance of the mechanism design, Bagnoli and McKee (1991) proved that a linear *VCM* can lead to the Pareto efficient equilibrium if there is a provision point and all the contributions are returned if the public good is not provided.

Several authors tried to explore the role of communication: Dawes, McTavish and Shaklee (1977) found a positive effect on contributions during *VCMs*. Later, Andreoni and Rao (2011) found similar results in dictator games. On this track, research extended her attention on expectations and beliefs. For instance, Fischbacher, Gächter and Fehr (2001) found evidence of conditional cooperation among the players of a public good game: subjects showed to be willing to match the contributions of the other players. According to this theory, de Oliveira, Croson and Eckel (2015) showed that conditional cooperators' behaviour may be poisoned by the presence of *"bad apples"* (i.e. free-riders).

2 Standard Voluntary Contribution Mechanism

This section describes an institution used in research on voluntary contributions to analyse the choice between a private good and a public one: see — for instance — Marwell and Ames (1979), Isaac and Walker (1984), Anderson et al. (2004). It is called *Voluntary Contribution Mechanism (VCM)*, and it is going to be the theoretical benchmark of my analysis.

Consider a society that lasts one period. It is characterized by N different players indexed by i, who are called to contribute to a public good with $MPCR = m \geq 0$. This parameter, the Marginal Per Capita Return, is the marginal rate of substitution between the private good and the public one; thus, it measures the private gain obtained by the decision maker by moving an additional unit of money from the private good to the public account.

Each player has an endowment $e_i \geq 0$ and chooses to contribute to the public good with an amount $x_i \in [0, e_i], i = 1, \ldots, N$. The game is simultaneous. Player i's profit is:

$$\Pi_i = e_i - x_i + m \sum_{j=1}^{N} x_j \tag{1}$$

which can be rephrased as:

$$\Pi_i = e_i + (m-1)\, x_i + m \sum_{j \neq i} x_j \tag{2}$$

Notice that if $m \geq 1$, Π_i is increasing in x_i. Therefore, the optimal contribution is $x_i^* = e_i$ regardless the other players' choices. The economic interpretation is straightforward: Player i prefers to contribute to the public good if her private marginal return is at least equal to her marginal cost of contribution, which is equal to 1. Thus, the optimal contribution x_i^* is defined as:

$$x_i^* \equiv \begin{cases} 0 & \text{if } m < 1 \\ e_i & \text{if } m \geq 1 \end{cases}$$

Instead, contribution is worthy from a social point of view if the overall marginal return is at least equal to the marginal cost of contribution: $mN \geq 1$. Here the positive externality caused by the public good is taken into account. Summing up, three scenarios are possible:

- if $m \leq \frac{1}{N}$, contribution is not desirable from both a social and a private point of view;
- if $\frac{1}{N} < m < 1$, contribution is socially desirable but privately not convenient;
- if $m \geq 1$, contribution is desirable from both a social and a private point of view.

For the following analysis, let me define:

· the "*lower threshold of contribution*" $\underline{m}$: the value of the *MPCR* such that if $m \leq \underline{m}$, then contribution is not worthy from a social point of view, while it is desirable otherwise. Thus — keeping m constant — if $\underline{m}$ increases, it is more likely that the public good is not social desirable. Here $\underline{m} = \frac{1}{N}$;

· the "*upper threshold of contribution*" $\overline{m}$: the value of the *MPCR* such that if $m \geq \overline{m}$, then contribution is worthy from a private point of view, while it is not desirable otherwise. Thus — keeping m constant — if $\overline{m}$ increases, it is less likely that the decision maker's optimal choice is to contribute to the public good. Since it coincides with the private marginal cost of contribution, here $\overline{m} = 1$.

3 The *VCM* with a pleasure function

Relying on the *warm-glow giving* theory by Andreoni (1990), let me assume that each subject is characterized by a private emotional reward entailed by contribution because of the personal satisfaction of helping other people. Then, Player i's profit can be modified as:

$$\Pi_i = e_i - x_i + m \sum_{j=1}^{N} x_j + R_i(x_i) \tag{3}$$

where $R_i(x_i)$ is the *pleasure function* of Player i.

Definition 3.1. The *pleasure function* $R_i(x_i)$ denotes Player i's satisfaction after a contribution of amount x_i to a public good.

From Equation 3 it is possible to see that Π_i is now formed by a *monetary* part and a *non-monetary* one. Given that, the choice of x_i^*, where x_i^* denotes the optimal contribution, is determined by both the entire value of the public good and the single contribution. This means that — according to Andreoni (1990) — Player i is an *impure altruistic* agent.

Set of Assumptions 1: $R(x)$ is characterized by the following properties:

1. $R(0) = 0$: the pleasure of contribution is 0 if there is no contribution at all;
2. $R(x)$ is C^2: this is a technical requirement;
3. $R(x)$ is *strictly increasing* in x: the higher is the amount of contribution, the higher is the pleasure of contribution;
4. $R(x)$ is *strictly concave* in x: the marginal pleasure of contribution is strictly decreasing in x. The agent feels her contribution to be more helpful when it is passing — for instance — from 10€ to 20€ rather than from 1000€ to 1010€;
5. $R'(0) \leq 1$: this requirement ensures to obtain non-negative values for the *thresholds of contribution*. It is not fundamental for the model to work, but it is helpful to get interesting results.

Assuming all the other hypothesis of the model are unchanged, the introduction of the pleasure function leads to the following results:

Proposition 3.1. Consider a Voluntary Contribution Mechanism with a pleasure function $R_i(x_i)$. Then the optimal contribution of Player i, i=1,...,N, is:

$$x_i^* \equiv \begin{cases} 0 & \text{if } m \leq 1 - R_i'(0) \\ {R_i'}^{-1}(1-m) & \text{if } 1 - R_i'(0) < m < 1 - R_i'(e_i) \\ e_i & \text{if } m \geq 1 - R_i'(e_i) \end{cases}$$

where $R_i'(t) = \frac{\partial R(x)}{\partial x}\Big|_{x=t}$.

Proof. The proof is available in Appendix A.1

Proposition 3.2. consider a Voluntary Contribution Mechanism with $MPCR = m$ and a pleasure function $R_i(x_i)$. Then the optimal contribution $x_i^*, i = 1\ldots, N$, is *increasing* in m.

Proof. The proof is available in Appendix A.2.

The introduction of the pleasure function causes several innovations: first of all, in contrast with the standard model — which presents a binary optimal solution — now the decision maker's optimal choice can be to contribute just partially to the public good. The advantage of this result consists in obtaining a theoretical model which better describes empirical data. Moreover, the optimal amount of contribution becomes smoothly increasing in m, consistently with the findings of Isaac and Walker (1984).

The upper threshold of contribution $\overline{m}$ shrinks to $\overline{m}_i = 1 - R_i'(e_i)$, making easier to observe total contribution to the public good. Notice that now the thresholds of contribution — depending on both the functional form of the pleasure function and the endowment — become a personal feature of the agent. In particular, by Assumption 1.4 richer people, assuming equal pleasure functions, show higher $\overline{m}_i$: poorer people will be more likely to invest all their resources in the public account.

Similarly to the upper threshold of contribution, $\underline{m}_i$ is adjusted by a *shifting factor* $[1 - R_i'(x_i^*)]$ with respect to the theoretical benchmark. The analytical derivation is available in Appendix A.3. Notice that its value depends on the optimal contribution, which — being x_i^* a private decision — is exogenous in this part of the analysis. According to the sign of the shifting factor, two cases should be considered:

- if $R_i'(x_i^*) \geq 1$, then $\underline{m}_i \leq 0$: contribution is always socially desirable if the non-monetary marginal benefit alone is already big enough to compensate the marginal cost, which is equal to 1 in this model;
- if $R_i'(x_i^*) < 1$, then $\underline{m}_i > 0$: there is a set of public goods that becomes socially desirable because of the presence of the non-monetary component of the private payoff.

Furthermore, a new threshold of contribution appears in the model. Let me call it *"threshold of partial contribution"* $\hat{m}_i$: it represents the value of the *MPCR* such that if $\hat{m}_i \leq m \leq \overline{m}_i$, then it is optimal to contribute just partially to the public account from a private point of view. Thus, the higher is $\hat{m}_i$, the less likely is contribution going to represent the optimal choice. Here, $\hat{m}_i = 1 - R_i'(0)$.

An example of a *VCM* with pleasure function $R(x) = \ln(x + 1)$ is available in Appendix A.4.

4 Impure altruism and group size

Let me now assume the pleasure function depends on the number of beneficiaries of the public good, $S \in \mathbb{N}$. The aim of this section is to define the relation of the voluntary contribution with this parameter.

From Section 1, it is possible to observe that results by previous experimental research on this correlation are ambiguous. For instance, Marwell and Ames (1979) tried to explore it using a *VCM* with a provision point: it means that there exists a minimum amount of total contributions that must be reached in order to provide the public good. Their results showed a negative influence of the group size on contributions, even if they were not statistically significant. Instead, Isaac and Walker (1988) found exactly the opposite relation introducing repetition and removing the provision point. The same finding was then obtained by Kahneman and Knetsch (1992). I believe this happens because of two main reasons:

- there is a conceptual difference between the beneficiaries of a public good, who do not compete among themselves because of the non-rivalry property, and the players of a *VCM*, who may be in a competitive environment, instead;
- experiments with a provision point and experiments without a provision point are not comparable since they represent two different theoretical games: *VCM* with provision point is a *chicken game*, while a *VCM* without it is a *prisoner's dilemma*.

The analysis of this essay is based on a *VCM* without provision point, with a constant Marginal Per Capita Return of the public good with respect to the size of the community.

In the light of these observations, the remainder of this section is organized as follows. In subsection 4.1 I discuss the properties of the pleasure function modified by S. In subsection 4.2 I apply this function to a particular type of *VCM*.

4.1 The pleasure function modified by S

Definition 4.1. the *pleasure function* $R_i(x_i, S)$ denotes Player i's satisfaction after a contribution of amount x_i to a public good with S beneficiaries.

Set of Assumptions 2: $R(x, S)$ is characterized by the following properties:

1. $R(0, S) = 0 \, \forall \, S \in \mathbb{N}$: the pleasure of contribution is zero if there is no contribution at all, regardless the number of beneficiaries;
2. $R(x, 0) = 0 \, \forall \, x \in [0, +\infty[$: the pleasure of contribution is zero if no one will consume the public good, regardless the amount of contribution;
3. $R(x, S)$ is C^2 with respect to x: this is a technical requirement;

4. $\forall\, S > 0,\ R(x,S)$ is *strictly increasing* in x : for a given positive number of beneficiaries, the higher is the amount of contribution, the higher is the pleasure of contribution;

5. $\forall\, x > 0,\ R(x,S)$ is *strictly increasing* in S: for a given positive contribution, the higher is the number of beneficiaries, the higher is the pleasure of contribution;

6. $R(x,S)$ is *strictly concave* in x: the marginal pleasure of contribution is strictly decreasing in x;

7. $\frac{\partial^2 R(x,S)}{\partial x \partial S} \geq 0\, \forall\, (x,S)$: the marginal pleasure of contribution is increasing in the number of beneficiaries. To get the intuition, suppose you have voluntarily given an amount x to university Federico II in order to build a fountain in Monte Sant'Angelo. Then they ask you to give an additional euro for the project. My claim is that you will be more willing to accept the request if the fountain is supposed to be built in the main entrance of Monte Sant'Angelo — where any student can admire it — rather than in the Biology department, where only Biology students can look at it;

8. $\forall\, S > 0,\ R'(0,S) \leq 1$: this requirement ensures to obtain non-negative values for the *thresholds of contribution.* It is not fundamental for the model to work, but it is helpful to get interesting results.

4.2 Altruistic *VCM* and number of beneficiaries

Assume the agent plays a *VCM* without provision point with N players. In this environment, all and only the participants are the recipients of the public good: $S = N$. Moreover, let me remark that the *MPCR* is constant with respect to N, according to the setting proposed by Isaac and Walker (1988). Then, the following results hold:

Proposition 4.1. Consider a Voluntary Contribution Mechanism with a pleasure function $R_i(x_i, N)$. Then the optimal contribution of Player i, $i = 1, \ldots, N$, is:

$$x_i^* \equiv \begin{cases} 0 & \textit{if } \ m \leq 1 - R_i'(0,N) \\ {R_i'}^{-1}(1-m, N) & \textit{if } \ 1 - R_i'(0,N) \ < m < \ 1 - R_i'(e_i, N) \\ e_i & \textit{if } \ m \geq 1 - R_i'(e_i, N) \end{cases}$$

where $R_i'(t,N) = \frac{\partial R(x,N)}{\partial x}\Big|_{x=t}$.

Proof. Omitted.[2]

Proposition 4.2. consider a Voluntary Contribution Mechanism with $MPCR = m$ and a pleasure function $R_i(x_i, N)$. Then the optimal contribution $x_i^*, i = 1, \ldots, N$, is *increasing* in m.

[2]The proof is equivalent to the one of Proposition 3.1.

Proof. Omitted.[3]

While Proposition 4.1 and Proposition 4.2 only confirm the results of Section 3, the next proposition describes the relation of the optimal contribution with the new parameter of the model, N.[4]

Proposition 4.3. consider a Voluntary Contribution Mechanism with a pleasure function $R(x, N)$. Then the optimal contribution x^* is *increasing* in N.

Proof. The proof is available in Appendix B.1

By Proposition 4.3, smaller groups should be characterized by higher levels of *free-riding*, consistently with the findings by both Isaac and Walker (1988) and Kahneman and Knetsch (1992).

Finally, let me discuss how the introduction of N modifies the *thresholds of contribution*. Both $\hat{m}_i = 1 - R_i'(0, N)$ and $\overline{m}_i = 1 - R_i'(e_i, N)$ negatively depends on N by Assumption 2.7: an increase in the group size lowers the thresholds for both partial and total contribution, which accords to Proposition 4.3.

On the other hand, the effect on $\underline{m}_i$ is ambiguous. In this formulation of the model, $\underline{m}_i = \frac{1}{N}\left[1 - R_i'(x_i^*, N)\right]$. Similarly to Section 3, $\underline{m}_i$ is shifted by a factor $[1 - R_i'(x_i^*, N)]$, which can lead to a trivial threshold or not depending on its sign (look at Section 3 for a discussion about the size of $R_i'(x_i^*, N)$ and the consequent economic interpretation). The analytical derivation of $\underline{m}_i$ is available in Appendix B.2. Notice the net effect of N on $\underline{m}_i$ is not predictable *a priori*, since N shows:

- a *negative direct effect* on $\underline{m}_i$ through $\frac{1}{N}$: as N increases, the overall marginal return of contribution increases, making contribution more convenient from a social point of view;
- an *uncertain indirect effect* on $\underline{m}_i$ through $R_i'(x_i^*, N)$. The ambiguity depends on the interaction between Assumption 2.6, Assumption 2.7 and Proposition 4.3. In fact, Assumption 2.6 and Proposition 4.3 imply that as N increases, $R_i'(x_i^*, N)$ decreases through the increase of x_i^*. Instead, Assumption 2.7 states that — all else equal — $R_i'(x_i^*, N)$ is increasing in N.

An example of a *VCM* with a pleasure function $R(x, N) = N\sqrt{x}$ is available in Appendix B.3.

[3]The proof is equivalent to the one of Proposition 3.2.

[4]To simplify the notation, I avoid to use the index i when it is not necessary.

5 Impure altruism and inequality

Let me now assume the pleasure function depends on the level of income inequality between the decision maker and the beneficiaries of the public good. This assumption is intuitive: I strongly believe you would be gladder to help Oliver Twist rather than Ebenezer Scrooge. Justifications of such behaviour come from two main fields:

- from Economics, by *law of diminishing marginal utility* it is possible to state that — all else equal — an additional unit of money has a greater impact in terms of additional utility on a poorer subject. Thus, if the decision maker cares about helping other people and wants to generate as much utility as possible, she will help poorer agents;

- from Ethics, helping poorer people has always been an extremely positive action. Robin Hood, the legendary heroic outlaw from English folklore, is a clear example of this phenomenon: his philosophy of life *"steal from the rich and give to the poor"* is so well-regarded that it compensates the fact that Robin Hood is a thief, making him a positive character.

This assumption has been largely confirmed by previous experimental research: for instance, look at Anderson et al. (2004, 2008) or Heap et al. (2016). Thus, the aim of this section is to add income inequality to the *altruistic VCM*, and to explore the effect of this qualitative difference among the beneficiaries on the voluntary contribution.

To reach this goal, the remainder of the section is organized as follows. In subsection 5.1 I discuss a reasonable measure to introduce inequality in the pleasure function. In subsection 5.2 I describe the properties of the pleasure function modified by inequality. In subsection 5.3 I focus on the *VCM* without provision point and constant *MPCR* with respect to N.

5.1 How to take inequality into account

Assume the subject must decide how much to contribute to a public good with S beneficiaries. Her income is $y_i \geq 0$. Moreover, assume she knows the vector of the **other** beneficiaries' incomes y, which — if the decision maker is included in the set of beneficiaries — is:

$$y = \begin{pmatrix} y_1, & \dots, & y_{S-1} \end{pmatrix}$$

Otherwise, it is:

$$y = \begin{pmatrix} y_1, & \dots, & y_S \end{pmatrix}$$

with $y_j \geq 0 \, \forall j$.

I choose to focus on the first case since it is the most reasonable assumption; moreover, it is the one used in *VCM.* To avoid trivial cases, assume $S \geq 2$. My claim is that, in order to make her choice, the decision maker compares y_i with all y_j, $j = 1, \dots, S-1$. The easiest two ways to make this comparison are:

- the *absolute difference*: the decision maker takes into account $(y_j - y_i)\,\forall j$;
- the *ratio*: the decision maker takes into account $\frac{y_j}{y_i}\,\forall j$.

I use the *ratio* for mathematical convenience.

Given this choice, let me define:

$$\mathbf{d_i} = \Big(d_{i1}, \quad \ldots, \quad d_{iS-1}\Big)$$

where $d_{ij} = \frac{y_j}{y_i}\,\forall j \neq i$. Clearly, d_{ij} is not defined if $y_i = 0$. However, this is not an issue since $y_i = 0$ is a trivial case where the feasibility constraint requires not to contribute.

5.2 The pleasure function modified by $\mathbf{d_i}$

Definition 5.1. the *pleasure function* $R_i(x_i, S, \mathbf{d_i})$ denotes Player i's satisfaction after a contribution of amount x_i to a public good with S beneficiaries whose income inequality is described by $\mathbf{d_i}$.

Set of Assumptions 3: the pleasure function $R(x, S, \mathbf{d})$ is characterized by the following properties:

1. $R(0, S, \mathbf{d}) = 0\,\forall\,(S, \mathbf{d})$: the pleasure of contribution is zero if there is no contribution at all, regardless the number of beneficiaries and their distribution of income;
2. $\lim_{d_j \to +\infty} R(x, S, \mathbf{d}) = 0\,\forall j$: if there is (at least) one beneficiary who is infinitely richer than the decision maker, then the pleasure of contribution will be zero regardless the size of the group and the amount of contribution. The decision maker has no incentive to help in this case, since that player does not appear deserving or needy;
3. $R(x, S, \mathbf{d})$ is C^2 with respect to x: this is a technical requirement;
4. $R(x, S, \mathbf{d})$ is *strictly increasing* in x: for a given set of beneficiaries, the higher is the amount of contribution, the higher is the pleasure of contribution;
5. $\frac{\partial R(x,S,\mathbf{d})}{\partial \mathbf{d}} \frac{\partial \mathbf{d}}{\partial S} = 0 \Rightarrow \frac{\partial R(x,S,\mathbf{d})}{\partial S} > 0$: if the effect of income inequality is kept constant, then the higher is the number of beneficiaries, the higher is the pleasure of contribution;
6. $R(x, S, \mathbf{d})$ is *strictly decreasing* in $d_j\,\forall j$: for each beneficiary, the pleasure of contribution is strictly decreasing in her relative income;
7. $R(x, S, \mathbf{d})$ is *strictly concave* in x: for a given set of beneficiaries, the marginal pleasure of contribution is strictly decreasing in x;
8. $\frac{\partial^2 R(x,S,\mathbf{d})}{\partial x \partial \mathbf{d}} \frac{\partial \mathbf{d}}{\partial S} = 0 \Rightarrow \frac{\partial^2 R(x,S,\mathbf{d})}{\partial x \partial S} \geq 0$: if the effect of income inequality is kept constant, the marginal pleasure of contribution is increasing in the number of beneficiaries;

9. $\frac{\partial^2 R(x,S,\mathbf{d})}{\partial x \partial d_j} \leq 0\ \forall j$: all else equal, the marginal pleasure of contribution is decreasing in the relative income of each beneficiary: the decision maker will be more willing to give an additional euro to the public good if the latter is supposed to help deserving people;

10. $R'(0, S, \mathbf{d}) \leq 1$: this requirement ensures to obtain non-negative values for the *thresholds of contribution*. It is not fundamental for the model to work, but it is helpful to get interesting results.

5.3 Altruistic *VCM* and inequality

Assume the agent plays a *VCM* without provision point with N players. In this environment, all and only the participants are the recipients of the public good: $S = N$. Moreover, remember that the *MPCR* is constant with respect to N. Then, the following results hold:

Proposition 5.1. Consider a Voluntary Contribution Mechanism with a pleasure function $R_i(x_i, N, \mathbf{d_i})$. Then the optimal contribution of Player i, i=1,...,N, is:

$$x_i^* \equiv \begin{cases} 0 & \text{if } m \leq 1 - R_i'(0, N, \mathbf{d_i}) \\ R_i'^{\,-1}(1-m, N, \mathbf{d_i}) & \text{if } 1 - R_i'(0, N, \mathbf{d_i}) < m < 1 - R_i'(e_i, N, \mathbf{d_i}) \\ e_i & \text{if } m \geq 1 - R_i'(e_i, N, \mathbf{d_i}) \end{cases}$$

where $R_i'(t, N, \mathbf{d_i}) = \frac{\partial R(x,N,\mathbf{d_i})}{\partial x}\big|_{x=t}$.

Proof. Omitted.[5]

Proposition 5.2. consider a Voluntary Contribution Mechanism with $MPCR = m$ and a pleasure function $R_i(x_i, N, \mathbf{d_i})$. Then the optimal contribution $x_i^*, i = 1, \ldots, N$, is *increasing* in m.

Proof. Omitted.[6]

Proposition 5.3. consider a Voluntary Contribution Mechanism with a pleasure function $R(x, N, \mathbf{d})$. Then the optimal contribution x^* is *decreasing* in $d_j\ \forall j$.

Proof. Omitted.[7]

Again, Proposition 5.1 and Proposition 5.2 only confirm the results of previous sections. The novelty of this section is represented by Proposition 5.3, which accords to the findings of past literature, as it is possible to learn from Section 1.

[5]The proof is equivalent to the one of Proposition 3.1.
[6]The proof is equivalent to the one of Proposition 3.2.
[7]The proof is equivalent to the one of Proposition 4.3.

Let us focus on the effects of income inequality on the *thresholds of contribution*. For this analysis, let us keep N constant. First of all, by Assumption 3.9 it is possible to observe that both $\hat{m}_i = 1 - R_i'(0, N, \mathbf{d_i})$ and $\overline{m}_i = 1 - R_i'(e_i, N, \mathbf{d_i})$ positively depend on $d_j \, \forall j = 1, \ldots, N-1$. Thus, by considering richer recipients, both partial and total contribution are less likely to be chosen for a given value of the *MPCR*. This result is consistent with the taste for *fairness* introduced in the model through $\mathbf{d_i}$.

About $\underline{m}_i$, its value is shifted by the factor $[1 - R_i'(x_i^*, N, \mathbf{d_i})]$ with respect to the theoretical benchmark. This result is equivalent to the ones of previous sections: thus, look at Section 3 for a discussion about the sign of the *shifting factor* and the economic intuition behind it. The analytical derivation of $\underline{m}_i$ is available in Appendix C.1

Moreover, notice that — similarly to the previous analysis on N — the effect of a variation of d_{ij} on $\underline{m}_i$ is ambiguous $\forall j$. In fact, by Assumption 3.9 it has a positive effect on $\underline{m}_i$ through $R_i'(x_i^*, N, \mathbf{d_i})$: for each recipient, an increase in income inequality with respect to the decision maker makes contribution less desirable also from a social point of view. On the other hand, by Proposition 5.3 an increase in d_{ij} causes a decrease in x_i^*, which — by assumption of strict concavity — has a positive effect on the marginal pleasure of contribution. Thus, it generates a decrease in $\underline{m}_i$. Again, the net effect is not predictable *a priori*.

An example of a *VCM* with a pleasure function $R(x, N, \mathbf{d}) = \frac{N\sqrt{x}}{\prod_{j=1}^{N-1} d_j + 1}$ is available in Appendix C.2.

6 The experimental designs

The following experimental designs are aimed at testing some results of the *altruistic VCM*. In particular, the experiments described in subsections 6.1 and 6.2 are conceived to explore the relation between the voluntary contribution to a public good and the number of its beneficiaries. On the other hand, the experiment described in subsection 6.3 is designed to analyse the relation of the voluntary contribution to a public good with the relative income of each other contributor.

6.1 The medical research experiment

The experiment is conceived to test if the voluntary contribution to a public good is increasing in the number of its beneficiaries.

Subjects are required to split 10 coins, each one with a value of 1€, between themselves and a research program on an infectious disease by the Department of Biology of Federico II. Notice that:

- research on infectious diseases is a public good since the knowledge deriving from it is both non-excludable and non-rival;
- since the Marginal Per Capita Return is almost zero, the theoretical, profit-maximizing strategy of this game is not to contribute to the research program. This happens because the subjects can assign to the public pot a quantity which is really small with respect to the necessary amount of money required to obtain relevant results in research on infectious diseases; thus, the gain deriving from an additional euro is almost null.

Each treatment is referred to a different pathology. Of each disease, the subjects know only the number of contagions N and the basic reproduction number R_0, which is *the expected number of cases directly generated by one case in a population where all the individuals are susceptible to infection.* Notice that both these parameters are a proxy for the number of beneficiaries of the research program. The design consists of two treatments[8] that vary both N and R_0:

Treatment #	Number of contagions	Reproduction number
Treatment 1 (T1)	$N = 37.900.000$	$R_0 = 5$
Treatment 2 (T2)	$N = 228.000.000$	$R_0 = 115$

[8] I choose only two treatments to minimize implementation costs.

The parameters used in Treatment 1 describe HIV[9], while the ones used in Treatment 2 describe Malaria.[10] Data about contagions are referred to year 2018, since they are the most recent available data from WHO.

The pathogens have been chosen because they present a deep difference in both N and R_0. In fact — although even a single unit matters from a mathematical point of view — subjects might not discriminate the treatments if the values are not different enough.

The name of the pathogen is voluntarily omitted to avoid data to be biased by an *affect heuristics*: subjects may contribute to a project just because they recognize the denomination of the disease and they are more willing to contribute to something they know. To get the intuition, assume I would have inserted a third treatment with these parameters:

Treatment #	Number of contagions	Reproduction number
Treatment 3 (T3)	$N = 10.842.028$	$R_0 = 3.5$

Since this treatment shows the smallest values of both N and R_0, we should expect the smallest contribution, too. However, I am sure this would not happen with the introduction of the name of the pathogen: these parameters, in fact, are referred to *SARS-CoV-2*.

To avoid experimenter effects, the experiment is run under *"double anonymity"*: each subject's choice is hidden to both the other subjects and the experimenter.[11] In this way, each individual is completely free to choose her contribution without the fear of being judged, neither by the other players nor by the experimenter.

The experiment presents a within-subjects design to compare the outcomes of (T1) and (T2): in each session, subjects play under both the treatments. Moreover, a between-subjects design is used to control *order effects*: individuals may be influenced by the order used to present the treatments. Thus, I would run 6 sessions overall, using (T1) as first treatment and (T2) as second one in 3 of them, reversing the order in the others. Each session lasts one period for each treatment, since the study is focused on subjects' behaviour in a one-shot environment.

For each session, 30 students are randomly chosen from the Economics Department of Federico II. Prior to the beginning of the experiment, every subject receives a show-up fee of 5€ and is endowed with instructions, which are available upon request.

Relying on the theoretical predictions of the *altruistic VCM*, contributions should be higher in (T2).

[9] R_0 estimation is from [34].
[10] R_0 estimation is from [38].
[11] The assumption is from [25].

6.2 The *Gemino* DG experiment

The experiment — like the previous one — is designed to test if the voluntary contribution to a public good is increasing in the number of its beneficiaries.

Subjects are called to split 10€ between themselves and a group of recipients. The novelty with respect to the standard Dictator Game consists in a variation of both the number of recipients and the multiplier of the donation, in order to modify the game as a public good setting. Previous research — as stated in Engel (2011) — has already explored the relation of these parameters with the Dictator's contribution by taking them just singularly. Both have shown a positive, significant influence.

In this design, these two features coincide according to the following mechanism: 1€ from the decision maker (who is called *splitter*) coincides with 1€ for each member of the recipients' group. This means that if the group has size 1, it will receive 1€, if it has size 3, it will receive 3€ and so on. In this way, each donated euro is both non-rival and non-excludable.

Notice that — under these conditions — the marginal cost of contribution to the group, which is equal to 1€ , is strictly greater than the private marginal benefit, which is equal to 0€: consequently, the theoretical, profit-maximizing strategy of the splitter is to keep all the money for herself.

The design consists of three treatments:

Treatment #	Number of recipients
Small size (S)	1
Medium size (M)	6
Large size (L)	12

Notice that (S), which is the control treatment, coincides with a standard Dictator Game: each donated euro is moved from the splitter to the recipient without any additional action of the experimenter. Instead, under treatment (M), the experimenter adds 5€ to each euro from the splitter, so that each recipient gets 1€. Similarly, under the treatment (L), the experimenter adds 11€ to each unit donated by the splitter.

The number of recipients of each treatment has been chosen in order to present a relevant difference in terms of group size: although even just one unit matters from a mathematical point of view, subjects might not discriminate the treatments if the dimensions of the groups are similar.

The experiment is run through computers: in this way, *experimenter effects* are reduced by avoiding face-to-face interaction. Moreover, divisors guarantee privacy to the subjects, hiding them from each other.

The experiment presents a within-subjects design to compare the outcomes over the treatments, while a between-subjects design is used to control *order effects*: subjects may act differently according to the order used to present the treatments. Thus, I would run 6 sessions overall, using the order (S), (M), (L) in 3 of them, reversing it in the others. Each session lasts 20 periods, so that each subject is going to play one period as splitter and other 19 periods as a recipient. The assignation to small group, medium group or large one is completely random, and is repeated at every round: in this way, I avoid the subjects to prefer a group for a sense of belonging. During her period, the splitter plays 3 times, one for each treatment.

The aggregate earnings are shown at the end of the session: this approach — allowing to hide the choice of each subject to the other players — involves that people are less scared to be judged and choose the splitting freely. Furthermore, it permits to avoid revenges or mimicking behaviours.

For each session, 20 students are randomly chosen from the Economics Department of Federico II. The experiment is programmed using Z-tree software. Prior to the beginning of the experiment, all subjects receive a show-up fee of 5€ and are endowed with instructions, which are available upon request.

According to the theoretical predictions of the *altruistic VCM*, contributions should increase with the number of recipients; thus, I should observe the highest contributions under the treatment (L), medium amounts under the treatment (M) and the smallest ones under the control (S). This result would be consistent with the findings of past research on both the number of recipients and the multiplier of the donation.

6.3 The inequality experiment

The experiment is conceived to test if the voluntary contribution to a public good is negatively related to the relative income of the other contributors.

Subjects are called to play a one-shot Voluntary Contribution Mechanism characterized by the following parameters: $N = 3$, $m = \frac{2}{3}$, $e_i = 10$ for all $i = 1, 2, 3$. Notice that — under these conditions — the Nash equilibrium requires to free-ride; however, cooperation would lead to higher profits.

Each time, the game is preceded by a *pre-stage phase*: after the subjects have been arranged in groups of 3 people, they may receive an amount of money called *premium*. Premia cannot be given to the public pot during the game: they are directly put in the private account.[12] This mechanism is used to introduce inequality in the game without changing the players' feasible set of actions. After the assignment of the premia, the subjects receive the endowment e_i and play. So, the income of each subject is given by the sum of both the premium and the endowment.

The design consists of three treatments varying the size of the premia:

[12] This assumption is from [2]

Amount of premia			
Treatment #	Player 1	Player 2	Player 3
Zero Winners (T0)	0	0	0
One Winner (T1)	0	0	7
Two Winners (T2)	0	7	7

In (T0), the control treatment, no one gets the premium, in (T1) only one player gets the premium, in (T2) two players get the premium. Its size is different from the endowment to avoid *experimenter demand effects*: subjects may try to figure out how the experimenter wants them to behave.

At each round, the players know each agent's endowment and how many people within the group are going to receive the premium: in other words, they know under which treatment they are going to play. This implies that each player knows exactly both her own income and the other players' ones; however, in contrast with the setting by Anderson et al. (2004), it is not relevant if they know the identity of the winner(s).

The experiment presents a within-subjects design to compare the treatments to the control: in each session, players face 10 periods of (T0) and then 10 periods of either (T1) or (T2), for a total of 20 periods. Moreover, a between-subjects design is developed to compare (T1) and (T2). After each round, the subjects are randomly re-arranged in new triples: in this way, each observation is independent from the other ones; the assignation of premia in (T1) and (T2) is completely random, too. Overall, I would run 3 sessions of (T0/T1) and 3 sessions of (T0/T2).

(T1) and (T2) are split to minimize *order effects*: individuals may be influenced by the order used to present the treatments. Moreover, this strategy allows to avoid to deal with tired agents. Payoffs are shown at the end of the session to avoid revenges or mimicking behaviours.

For each session, 24 students are randomly chosen from the Economics Department of Federico II. The experiment is programmed using Z-tree software. Prior to the beginning of the experiment, all subjects receive a show-up fee of 5€ and are endowed with instructions, which are available upon request.

On the basis of the theoretical analysis, Player 1's contribution should decrease over the treatments, Player 2 should contribute less in (T1) than in (T0) and more in (T2), Player 3 should contribute more in both (T1) and (T2), but the contribution should be smaller in (T2) than in (T1).

7 Final considerations

The aim of this section is to summarize the main innovations of the *altruistic VCM*, and to express some final considerations about the results of the model.

In this work, *VCM* without provision point is upgraded with the introduction of some new features in the theoretical formulation, whose significant influence on voluntary contributions has been already shown by past research. In particular, I take into account:

- the *warm-glow* deriving from contribution;
- the number of beneficiaries N;
- the income inequality among players described by $\mathbf{d_i}$.[13]

These variables are embedded in the model through a *pleasure function* $R_i(x_i, N, \mathbf{d_i})$, which represents a non-monetary reward in the decision maker's payoff.

The effect of this modification of the *VCM* is an increase in the reliability of the theoretical predictions, which are now characterized by the following properties:

1. the optimal strategy can be to contribute, even partially, to the public good;
2. the optimal contribution x_i^* is smoothly increasing in the *MPCR*;
3. all else equal, the optimal contribution x_i^* is increasing in N;
4. all else equal, the optimal contribution x_i^* is decreasing in d_{ij} $\forall j$.

Notice that Result 3 and Result 4 imply that **both the quantitative and qualitative compositions of the group matter**: an increase in the number of beneficiaries of the public good does not always generate an increase in the pleasure of contribution, since it depends on the *type* of recipients that are added to the community.

Clearly, this has an effect on the optimal contribution x_i^*. In particular, if an infinitely rich recipient is introduced in the community, then the model will degenerate to the standard *VCM* by Assumption 3.2. Thus, assuming $m < 1$, $x_i^* = 0$. This suggests that — as suggested by De Oliveira et al. (2015) — **one bad apple can spoil the whole bunch**.

[13] Remember that $\mathbf{d_i} = \begin{pmatrix} d_{i1}, & \dots, & d_{iN-1} \end{pmatrix}$, with $d_{ij} = \frac{y_j}{y_i}$ $\forall j \neq i$.

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A Section 3 — Appendix

A.1 Proposition 3.1 — Proof

Each Player i solves:

$$\sup_{0 \le x_i \le e_i} \Pi_i = e_i - x_i + m \sum_{j=1}^{N} x_j + R_i(x_i) \tag{4}$$

Notice that:

- Π_i is a composition of continuous functions; thus, it is continuous;
- $[0, e_i]$ is a closed and bounded subset of $\mathbb{R}$, which is a finite dimensional space endowed by hypothesis with the Euclidean norm; so, the feasible set is compact.

Thus, Problem 4 admits at least one solution by Weierstrass' theorem. Rearranging the objective function, the problem becomes:

$$\max_{0 \le x_i \le e_i} \Pi_i = e_i + (m-1)\, x_i + m \sum_{j \ne i} x_j + R_i(x_i) \tag{5}$$

Since neither the endowment nor the remaining value of the public good depend on the choice variable, (5) can be simplified in:

$$\max_{0 \le x_i \le e_i} \Pi_i = (m-1)\, x_i + R_i(x_i) \tag{6}$$

Three different cases are possible: $x_i^* = 0$, $x_i^* = e_i$, x_i^* is an interior point.

1. **Zero contribution:** the solution of Problem 6 is zero contribution for sure if Π_i is decreasing in x_i. This happens when:
$$\frac{\partial \Pi_i}{\partial\, x_i} \;=\; m - 1 + R_i'(x_i) \le\; 0 \,\forall\, x_i \in [0, e_i]$$
By hypothesis of strict concavity, $R_i'(x_i)$ is strictly decreasing. This implies $R_i'(0) \ge R_i'(x_i)\, \forall x_i \in [0, e_i]$. Consequently, the previous inequality is satisfied if $m \le 1 - R_i'(0)$.

2. **Total contribution:** the solution of Problem 6 is total contribution for sure if Π_i is increasing in x_i. This happens when:
$$\frac{\partial \Pi_i}{\partial\, x_i} \;=\; m - 1 + R_i'(x_i) \ge\; 0 \,\forall\, x_i \in [0, e_i]$$
By hypothesis of strict concavity, $R_i'(x_i)$ is strictly decreasing. This implies $R_i'(e_i) \le R_i'(x_i) \,\forall\, x_i \in [0, e_i]$. Consequently, the previous inequality is satisfied if $m \ge 1 - R_i'(e_i)$.

3. **Partial contribution:** in the remaining interval, the solution of Problem 6 is found by Fermat's theorem:
$$\frac{\partial \Pi_i}{\partial\, x_i} \;=\; m - 1 + R_i'(x_i) \;=\; 0$$

which leads to:

$$x_i^* = {R'}_i^{-1}(1-m)$$

Notice that:

- Π_i is a composition of concave functions by construction, thus this necessary condition is also sufficient;
- $R_i'(\cdot)$ is invertible by strict monotonicity. ■

A.2 Proposition 3.2 — Proof

Consider a *VCM* with a pleasure function $R_i(x_i)$. According to Proposition 3.1, three cases should be considered:

- if $m \leq 1 - R_i'(0) \Rightarrow x_i^* = 0$. In this case the optimal contribution is constant;
- if $m \geq 1 - R_i'(e_i) \Rightarrow x_i^* = e_i$. In this case the optimal contribution is constant;
- if $1 - R_i'(0) < m < 1 - R_i'(e_i) \Rightarrow x_i^* = {R'}_i^{-1}(1-m)$. Notice that by Assumption 1.4 $R_i'(x_i)$ is *strictly decreasing* in x_i. By property of inverse functions, this monotonicity is preserved by the inverse, meaning that ${R'}_i^{-1}(x_i)$ is *strictly decreasing* in x_i too. Thus, as m increases, since $(1-m)$ strictly decreases, $x_i^* = {R'}_i^{-1}(1-m)$ *strictly increases* in m.

Since x_i^* is either constant or strictly increasing in m in each interval, it is possible to state that x_i^* is *increasing* in m. ■

A.3 Derivation of $\underline{\mathbf{m}}$ with $R(x)$

The general approach to compute $\underline{m}$ consists in comparing the social marginal revenue of contribution to the public account to its social marginal cost.
Assume the decision maker is endowed with a pleasure function $R(x)$.[14] Then:

- the social marginal revenue of contribution is $R'(x^*) + mN$;
- the social marginal cost of contribution is 1.

Thus, contribution is socially desirable if:

$$R'(x^*) + mN \geq 1$$

which leads to:

$$m \geq \frac{1}{N} - \frac{R'(x^*)}{N}$$

This implies:

$$\underline{m} = \frac{1}{N} - \frac{R'(x^*)}{N}$$

[14] To simplify the notation, I avoid to use the index i when it is not necessary.

B Section 4 — Appendix

B.1 Proposition 4.3 — Proof

Assume the initial value of N is N_1. By Proposition 4.1, three cases should be considered:

- if $m \leq 1 - R'(0, N_1) \Rightarrow x^*(N_1) = 0$. Suppose to increase the size of the group from N_1 to N_2. By Assumption 2.7, $R'(0, N)$ is increasing in N: $\hat{m}$ decreases. Then, two cases may occur:

  · if $m \leq 1 - R'(0, N_2) \Rightarrow x^*(N_2) = 0$;

  · if $m > 1 - R'(0, N_2) \Rightarrow x^*(N_2) > 0$.

  This means x^* is increasing in N in this interval;

- if $1 - R'(0, N_1) < m < 1 - R'(e, N_1) \Rightarrow x^*(N_1)$ is an interior point. Suppose to increase the size of the group from N_1 to N_2. By Assumption 2.7, $R'(0, N)$ is increasing in N: both $\hat{m}$ and $\overline{m}$ decrease. Then, two cases may occur:

  · $m \geq 1 - R'(e, N_2) \Rightarrow x^*(N_2) = e$;

  · $1 - R'(0, N_2) < m < 1 - R'(e, N_2) \Rightarrow x^*(N_2)$ is implicitly defined by the first order necessary condition of the following optimization problem:

  $$\max_{0 \leq x \leq e} \Pi = (m-1)x + R(x, N_2)$$

  which is:

  $$\frac{\partial \Pi}{\partial x} = m - 1 + R'(x, N_2) = 0 \tag{7}$$

  Let me call $f(x, N) = m - 1 + R'(x, N)$. Notice that:

  * f is C^1 because it is a composition of a linear function $h(m) = m - 1$, that is C^∞, and the derivative of a C^2 function, which is C^1;
  * $f(x^*, N_2) = 0$ by definition of x^*;
  * $\frac{\partial f}{\partial x} = \frac{\partial^2 R(x,N)}{\partial_2 x} < 0$ by Assumption 2.6.

  Then, by Implicit Function Theorem:

  $$\frac{\partial x^*(N)}{\partial N} = -\frac{\frac{\partial^2 R(x^*, N_2)}{\partial x \partial N}}{\frac{\partial^2 R(x^*, N_2)}{\partial_2 x}} \geq 0$$

  Thus, x^* is increasing in N in this interval.

  Alternatively, it is possible to prove the same statement as follows: from Equation 7, x^* can be expressed as:

  $$x^* = R'^{-1}(1 - m, N_2)$$

Notice that $R'(x, N)$ is invertible by strict monotonicity (Assumption 2.6). By Assumption 2.7, $R'(x, N)$ is increasing in N. By property of inverse functions, the monotonicity in preserved by the inverse. Thus $R'^{-1}(x, N)$ is increasing in N, too.

· if $m \geq 1 - R'(e, N_1) \Rightarrow x^* = e$. Suppose to increase the size of the group from N_1 to N_2 Again, by Assumption 2.7, $R'(e, N)$ is increasing in N: $\overline{m}$ decreases. Since $m \geq 1 - R'(e, N_1)$, *a fortiori* $m \geq 1 - R'(e, N_2)$. Thus, $x^*(N_2) = e$. So x^* is constant in this interval.

Since $x^*(N)$ is either increasing or constant in N $\forall m \geq 0$, it is possible to conclude that $x^*(N)$ is *increasing* in N. ■

B.2 Derivation of $\underline{\mathbf{m}}$ with $R(x, N)$

Assume the decision maker is endowed with a pleasure function $R(x, N)$.[15] Then:

- the social marginal revenue of contribution is $R'(x^*, N) + mN$;
- the social marginal cost of contribution is 1.

Thus, contribution is socially desirable if:

$$R'(x^*, N) + mN \geq 1$$

which leads to:

$$m \geq \frac{1}{N} - \frac{R'(x^*, N)}{N}$$

This implies:

$$\underline{m} = \frac{1}{N} - \frac{R'(x^*, N)}{N}$$

C Section 5 — Appendix

C.1 Derivation of *VCM* with $R(x, N, \mathbf{d})$

Assume the decision maker is endowed with a pleasure function $R(x, N, \mathbf{d})$.[16] Then:

- the social marginal revenue of contribution is $R'(x^*, N, \mathbf{d}) + mN$;
- the social marginal cost of contribution is 1.

Thus, contribution is socially desirable if:

$$R'(x^*, N, \mathbf{d}) + mN \geq 1$$

[15] To simplify the notation, I avoid to use the index i when it is not necessary.

[16] To simplify the notation, I avoid to use the index i when it is not necessary.

which leads to:

$$m \geq \frac{1}{N} - \frac{R'(x^*, N, \mathbf{d})}{N}$$

This implies:

$$\underline{m} = \frac{1}{N} - \frac{R'(x^*, N, \mathbf{d})}{N}$$