Central Bank Digital Currency: Implications for Monetary Policy and Financial Stability

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October 26, 2022

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Intro

Introduction

“The G7 is launching a set of public policy principles for Retail CBDCs.”

“Unlike most of the digital money people use daily today, it would be issued directly by a central like the Bank of England in the UK.”

“New joint task force between the Treasury and the Bank of England to look into a potential CBDC as a complement to cash in bank deposits.”

Broader Interest in CBDCs

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Question: What are the financial stability and monetary policy implications of retail central bank digital currencies?

Thesis Contribution

A model to examine **retail CBDCs** which is a household (renumerated) account at the central bank.

Novel combination of channels examining the tradeoff between monetary policy transmission and financial fragility through the introduction of CBDCs.

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CBDCs provide an outside option to households and disciplines the market power of the banking sector, achieving the perfect competition outcome.

Central bank gains greater transmission of monetary policy. However, the reduction in aggregate profitability and increased default of the banking sector increases financial instability through multiple channels.

Model Setup

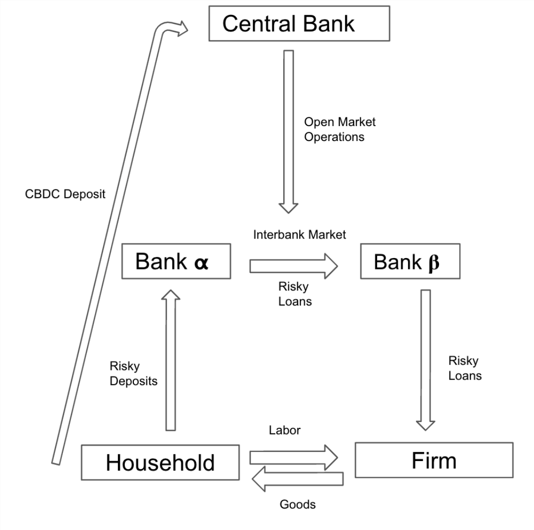

Figure 1: General Equilibrium.

Model Timeline

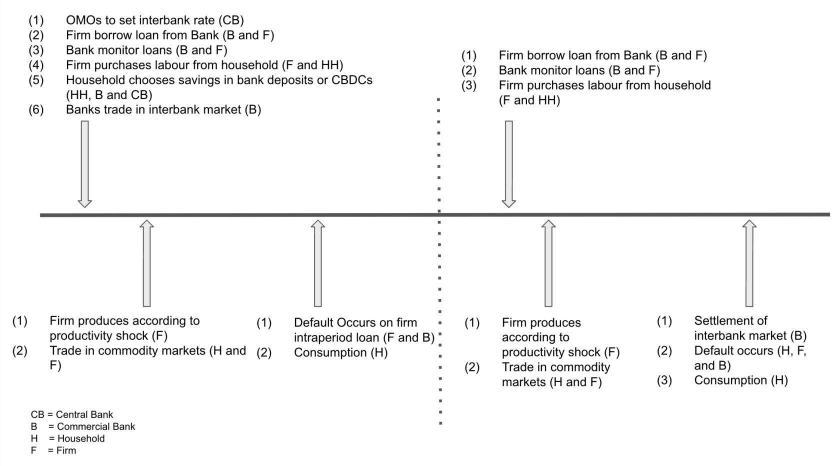

Figure 2: Model Timeline.

Household

Household Problem

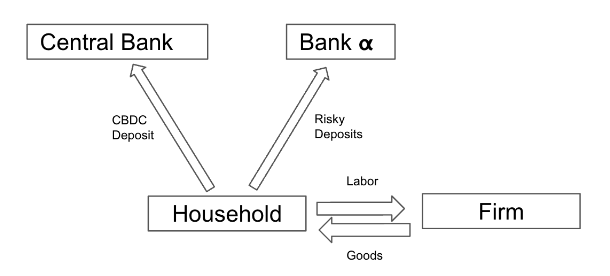

Figure 3: Household Sector.

Representative Household chooses intratemporal problem between consumption C_s^H and labour N_s^H.

Household Problem

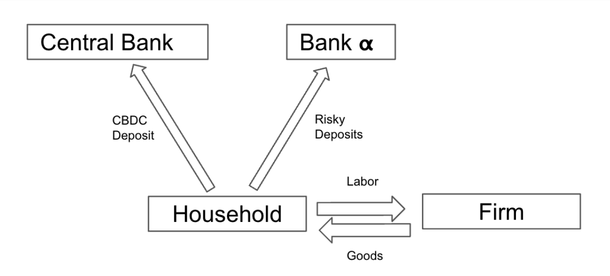

Figure 3: Household Sector.

Representative Household chooses intratemporal problem between consumption C_s^H and labour N_s^H.

Household also solves intertemporal consumption smoothing problem by investing in either (risky) bank deposits D^H at commercial bank α or central bank digital currencies Z^H at the central bank.

Household Objective

Household intertemporal objective:

$$\max_{\{C_s^H, N_s^H\}_{s \in \bar{\mathbb{S}}}, D^{H\alpha}, Z^H} \Pi^H = u(C_0^H, N_0^H) + \beta^H \sum_{s \in \mathbb{S}} \pi(s) u(C_s^H, N_s^H) \tag{1}$$

1. C_s^H: Consumption by household H in state s.
2. N_s^H: Labor supplied by household H to firm in state s.

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5. $D^{H\alpha}$: Commercial bank α deposit held by household H.
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Functional form of utility function:

$$u(C_s^H, N_s^H) = log[C_s^H] - \frac{N_s^{1+\psi}}{1+\psi} \quad (2)$$

Household Constraints in First Period

Subject to:

$$D^{H\alpha} + Z^{H} \leq W_0 N_0^{H} \tag{3}$$

Bank Deposits + CBDC $\leq$ Wage from Labour.

$$C_0^{H} \leq \frac{\Delta(3)}{p_0} \tag{4}$$

Consumption $\leq$ Amount of fiat money offered for good divided by price level (Shubik and Wilson 1977).

Household Constraints in Second Period

$$0 \leq W_s N_s^H + (1 + r^D) R_s^\alpha D^{H\alpha} + (1 + r^Z) Z^H, \quad \forall s \in \mathbb{S} \tag{5}$$

Wage from labour plus gross return on bank deposits and CBDCs where $R_s^\alpha \in [0, 1]$ is the fraction of bank deposits repaid by bank α.

$$C_s^H \leq \frac{\Delta(5)}{p_s}, \quad \forall s \in \mathbb{S} \tag{6}$$

Consumption $\leq$ Amount of fiat money offered for good divided by price level (Shubik and Wilson 1977).

Household Optimality Conditions

Intratemporal Consumption-Labour Euler Equation:

$$N_s^{\psi} C_s^{H} = \frac{w_s}{p_s}, \quad \forall s \in \overline{\mathbb{S}} \tag{7}$$

Household Optimality Conditions

Intertemporal Consumption Smoothing via Bank Deposits:

$$\mathbb{E}\left[\beta^H \frac{C_0^H p_0}{C_s^H p_s} R_s^\alpha\right] = \frac{1}{(1 + r^D)} \tag{8}$$

Intertemporal Consumption Smoothing via CBDC:

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Proposition

An interior solution for bank deposits exists if and only if:

$$r^Z < r^D.$$

Firms

Firm Problem

Firms face a cash-in-advance constraint to pay workers. Firms therefore drawn down bank credit lines from commercial bank β to pay workers.

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Finally, firms may choose to **endogenously default** on bank loans if the marginal benefit to defaulting is greater than the marginal cost (Dubey et al. 2005).

However, firms face a reputation penalty $\phi_s^F[\mathbb{I}_s]^+$ for defaulting where ϕ_s^F is the default penalty which is a proxy for firm's reputation (Wang 2022).

Firm Problem

Firms are **risk-averse** maximise 2-period profits Π^F:

$$\max_{\{N_s^F, B_s^{F\beta}, \nu_s^F\}_{s\in\bar{\mathbb{S}}}} \Pi^F = \frac{(\Pi_0^F)^{1-\gamma^F}}{1-\gamma^F} - \phi_0^F[\mathbb{I}_0^F]^+ + \sum_{s\in\mathbb{S}} \pi(s)\mathcal{M}_s^F \left\{ \frac{(\Pi_s^F)^{1-\gamma^F}}{1-\gamma^F} - \phi_s^F[\mathbb{I}_s^F]^+ \right\}$$

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where:

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3. $B_s^{F\beta}$: Loan from commercial bank β.

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4. $\nu_s^F \in [0,1]$: Fraction of bank loan that firm F pays back to commercial bank β.
5. $\phi_s^F[\mathbb{I}_s^F]^+$: Penalty for defaulting on loan to commercial bank β.

Firm Period 0 Constraints

Subject to:

$$W_s N_s^F \leq \frac{B_s^{F\beta}}{(1 + r_s^F)}, \quad \forall s \in \overline{\mathbb{S}} \tag{10}$$

Wage Paid to Labour $\leq$ Bank Credit Line.

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$$Y_0 = N_s^F \tag{11}$$

Firms produces output according to linear technology in period 0.

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Firms produces output according to linear technology in period 0.

$$\Pi_0^F = p_0 Y_0 - \nu_0^F B_0^{F\beta} \tag{12}$$

Firm's profit function is the revenue from producing the good plus leftover credit less the amount the firm decides to pay back on the loan $\nu_s^F \in [0,1]$.

Firm Period 1 Constraints

$$A_s \in \{A_b, A_g\} \tag{13}$$

Total factor productivity shock drawn in period 1.

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Default Penalty

$$[\mathbb{I}_s]^+ = \begin{cases} \left((1-\nu_s^F)B_s^{F\beta}\right)^2 & \text{if } \nu_s^F \in [0,1) \\ 0 & \text{if } \nu_s^F = 1 \end{cases}$$

The penalty to firm for defaulting is given by a quadratic cost (Tsomocos 2003).

Firm's Optimality Conditions

Proposition

Firm's optimal **default** condition is:

$$\left(\Pi_s^F\right)^{-\gamma^F} = 2\phi_s^F(1-\nu_s^F)B_s^{F\beta}, \quad \forall s \in \overline{\mathbb{S}} \tag{16}$$

Marginal Benefit to default = Marginal Cost to default.

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Proposition

Firm's optimal **borrowing** condition is:

$$\left(\frac{p_s A_s}{w_s(1+r_s^F)}\right)\left(\Pi_s^F\right)^{-\gamma^F} = 2\phi_s^F(1-\nu_s^F)B_s^{F\beta}, \quad \forall s \in \overline{\mathbb{S}} \tag{17}$$

Marginal Benefit to borrowing = Marginal Cost to borrowing.

Firm's Optimality Conditions

Proposition

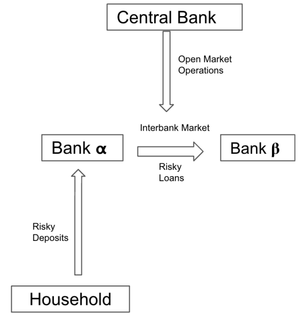
The wages paid are:

$$w_0 = \frac{p_0}{(1 + r_0^F)} \tag{18}$$

$$w_s = \frac{p_s A_s}{(1 + r_s^F)}, \quad \forall s \in \mathbb{S} \tag{19}$$

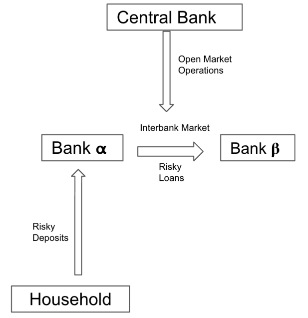
Banking

Bank α Problem



Bank α accepts deposits from household and issues interbank loans to bank β at the interbank market rate ρ.

Bank α Problem



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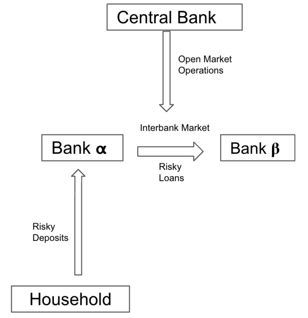

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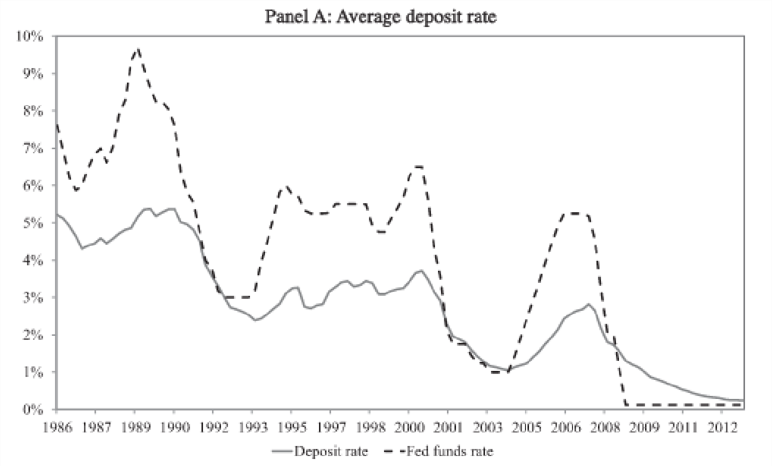
Bank α may endogenously default on the household deposit.

Deposit Bank α

Banks exhibit market power in deposit market (Dreschler et al. 2017).

Monopsony behaviour by keeping deposit rate below the federal funds rate.



Deposit Bank α Problem

Bank α maximises period 2 profits:

$$\max_{D^\alpha, L^{\alpha\beta}, \{\nu_s^\alpha\}_{s\in\mathbb{S}}} \quad \Pi^\alpha = \sum_{s\in\mathbb{S}} \pi(s)\left[\Pi_s^\alpha - \phi_s^\alpha[\mathbb{I}_s^\alpha]^+\right] \tag{20}$$

1. D^α: (Defaultable) household deposits.
2. ν_s^α : Fraction of deposits that is repaid to household.
3. $L^{\alpha\beta}$: (Risky) Interbank loans to bank β.
4. $\phi_s^\alpha[\mathbb{I}_s^\alpha]^+$: Deposits default penalty.

Deposit Bank α Constraints

Subject to:

$$L^{\alpha\beta} \leq D^{\alpha} \tag{21}$$

Bank Loans extended $\leq$ Household Deposits taken in.

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$$\Pi_s^{\alpha} = R_s^{\beta}(1+\rho)L^{\alpha\beta} - \nu_s^{\alpha}(1+r^D)D^{\alpha}, \quad \forall s \in \mathbb{S} \tag{22}$$

Profit is the spread between (repaid) loans and deposit where R_s^{β} is the fraction of loan repaid by bank β.

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Due to monopsony power, the quantity of deposits affects the deposit rate r^D:

$$r^D \equiv r^D(D^{\alpha})$$

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Loans are extended at an interbank market rate ρ influenced by the central bank and market forces.

Deposit Bank α Optimality Condition

Define the **supply elasticity of deposits**

$$\epsilon^D \equiv \frac{1 + r^D}{D^\alpha} \cdot \frac{\partial D^\alpha}{\partial r^D} \tag{23}$$

where $\epsilon^D > 0$.

Deposit Bank α Optimality Condition

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where $\epsilon^D > 0$.

Proposition

Due to market power and possibility of default, bank α sets a **markdown** on the deposit rate:

$$(1+r^D) = \frac{(1+\rho)}{(1+\frac{1}{\epsilon^D})} \frac{\mathbb{E}\left[R_s^\beta\right]}{\mathbb{E}\left[\nu_s^\alpha\right]} - \frac{\mathbb{E}\left[2\phi_s^\alpha(1-\nu_s^\alpha)^2 D^\alpha\right]}{\mathbb{E}\left[\nu_s^\alpha\right]} \tag{24}$$

Bank α restricts the supply of deposits to pay savers a lower deposit rate compared to the interbank market rate.

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Bank α restricts the supply of deposits to pay savers a lower deposit rate compared to the interbank market rate.

As $\epsilon^D \to \infty$, we arrive at **perfect competition**.

Proposition

Due to bank α market power, there is a dampening of monetary policy transmission via the **standard deposit channel** as the central bank adjusts the interbank rate ρ:

$$\frac{\partial r^D}{\partial \rho} < 1. \tag{25}$$

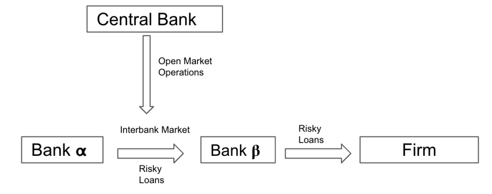
Bank α Default

Proposition

Bank α optimal default condition on household's deposit:

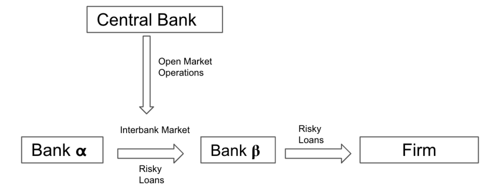
$$(1 + r^D) = 2\phi_s^\alpha (1 - \nu_s^\alpha) D^\alpha, \quad \forall s \in \mathbb{S} \tag{26}$$

Lender Bank β Borrowing



Bank β chooses the amount of interbank loans to borrow from bank α at the interbank rate ρ.

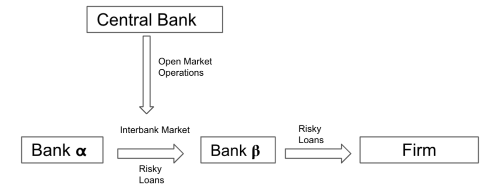
Lender Bank β Borrowing



Bank β chooses the amount of interbank loans to borrow from bank α at the interbank rate ρ.

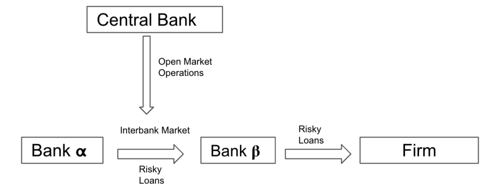
Bank β can endogenously default on the interbank loan.

Lender Bank β Lending



Bank β extends risky loans $L_s^{\beta F}$ to the firm, which the firm may default on. This introduces a **moral hazard problem** (Christiano and Ikeda 2013).

Lender Bank β Lending



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The bank can expend effort $M_{F,S}$ to monitor the credit worthiness of the firm and reduce likelihood of default by firm through imposing a higher default penalty cost.

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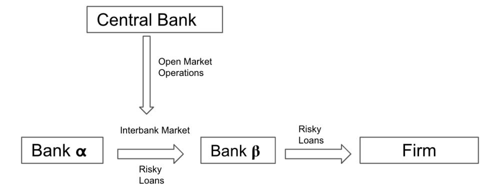

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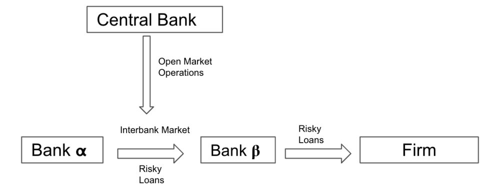
Monitoring firm is quadratic cost for bank (Martinez-Miera and Repullo 2017).

Lender Bank β Lending



Bank β extends risky loans $L_s^{\beta F}$ to the firm, which the firm may default on. This introduces a **moral hazard problem** (Christiano and Ikeda 2013).

The bank can expend effort $M_{F,s}$ to monitor the credit worthiness of the firm and reduce likelihood of default by firm through imposing a higher default penalty cost.

Monitoring firm is quadratic cost for bank (Martinez-Miera and Repullo 2017).

The bank internalises the firm's optimal default condition.

Lender Bank β Problem

Bank β is risk-neutral and maximises intertemporal profits:

$$\max_{\{L_s^{\beta F}, M_{F,s}\}_{s\in\bar{\mathbb{S}}}, \{\nu_s^{\beta}\}_{s\in\mathbb{S}}, B^{\beta\alpha}} \Pi^{\beta}$$

$$\Pi^{\beta} = \Pi_0^{\beta} - \frac{\gamma^F}{2} M_{F,0}^2 L_0^{\beta F} + \sum_{s\in\mathbb{S}} \pi(s)\mathcal{M}_s^{\beta} \left\{ \Pi_s^{\beta} - \frac{\gamma^F}{2} M_{F,s}^2 L_s^{\beta F} - \phi_s^{\beta} [\mathbb{I}_s^{\beta}]^{+} \right\}$$

where

1. $L_s^{\beta F}$: Loan from bank β to firm.
2. $B^{\beta\alpha}$: Interbank loan from bank α.

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1. $L_s^{\beta F}$: Loan from bank β to firm.
2. $B^{\beta\alpha}$: Interbank loan from bank α.
3. $M_{F,s}$: Amount of effort exerted to monitor firm.
4. ν_s^{β}: Fraction of loan to bank α that is repaid.
5. $\phi_s^{\beta}[\mathbb{I}_s^{\beta}]^+$: Default penalty for bank β defaulting on bank α.

Lender Bank β Period 0s Constraints

Subject to:

$$L_0^{\beta F} \leq \frac{B^{\beta\alpha}}{1+\rho} \tag{27}$$

Risky loan to firm $\leq$ interbank loan from bank α.

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The default penalty to the firm on the loan is equal to the monitoring effort by the bank (Wang 2022).

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$$\mathcal{M}_0^F = 2\phi_0^F(1-\nu_0^F)L_0^{\beta F} \tag{29}$$

Bank β internalises firm's optimal default condition.

Lender Bank β Period 0 Constraints

$$\Pi_0^\beta = (1 + r_0^H)L_0^{\beta H} + (1 + r_0^F)R_0^F L_0^{\beta F} \tag{30}$$

Bank β profit is the return on household's credit lines plus the loans to the firm that are paid back.

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$$e^\beta = \Pi_0^\beta \tag{31}$$

Bank β equity is the profit from first period.

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$$L_s^{\beta F} + \nu_s^{\beta} B^{\beta\alpha} \leq e^{\beta}, \quad \forall s \in \mathbb{S} \tag{32}$$

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$$\Pi_s^{\beta} = (1 + r_s^H) L_s^H + (1 + r_s^F) R_s^F L_s^{\beta F} - \nu_s^{\beta} B^{\beta\alpha}, \quad \forall s \in \mathbb{S} \tag{34}$$

Bank β profit is the return on household's credit lines plus the loans to the firm that are paid back less the amount that bank β decides to pay back on interbank loan.

Bank β Optimality Conditions

Proposition

Bank β Monitoring in Period 0:

$$\gamma^F M_{F,0}^2 = B^{\beta\alpha}(1+\rho)\mathbb{E}\left[\pi(s)\phi_s^\beta(1-\nu_s^\beta)\right]\left[1-\frac{\gamma_0^F(1+r_0^F)}{(1+r_0^H)}\right] \tag{35}$$

Interbank Rate

Proposition

The interbank rate ρ is given by:

$$(1+\rho) = \frac{B^{\beta\alpha}}{L^{\alpha\beta} + M^{CB}} \tag{36}$$

where $B^{\beta\alpha}$ is the interbank loan demanded by bank β and $L^{\alpha\beta}$ are the interbank deposits supplied by bank α and M^{CB} is the liquidity injected by central bank.

Lender Bank β Optimality Condition

Proposition: Bank Lending Channel

Bank β extends more risky loans to firms as they borrow more from the interbank market:

$$\frac{\partial L_s^{\beta F}}{\partial B^{\beta\alpha}} > 0. \tag{37}$$

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Proposition: Risk-Taking Channel

Bank β exerts less effort into monitoring loans as the interbank market rate ρ decreases:

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Bank β extends a higher quantity of risky loans as bank α supplies more loans into the interbank market.

Equilibrium Conditions

Market Clearing

There are 11 markets in the model. Let $\overline{\mathbb{S}} = \{0\} \cup \{g, b\}$.

Goods market:

$$C_s^H = Y_s, \quad \forall s \in \overline{\mathbb{S}} \tag{39}$$

Labor market:

$$N_s^F = N_s^H, \quad \forall s \in \overline{\mathbb{S}} \tag{40}$$

Firm credit loan market:

$$\frac{B_s^{F\beta}}{(1 + r_s^F)} = L_s^{\beta F}, \quad \forall s \in \overline{\mathbb{S}} \tag{41}$$

Bank deposit market:

$$(1+r^D)D^{H\alpha} = D^\alpha \tag{42}$$

CBDC market:

$$(1+r^Z)Z^H = Z^{CB} \tag{43}$$

Rational expectations:

$$R_s^\alpha = \nu_s^\alpha, \quad \forall s \in \mathbb{S} \tag{44}$$

$$R_s^\beta = \nu_s^\beta, \quad \forall s \in \mathbb{S} \tag{45}$$

$$R_s^F = \nu_s^F, \quad \forall s \in \overline{\mathbb{S}} \tag{46}$$

Prices

Goods prices:

$$p_s = \frac{M_s^H}{Y_s^F}, \quad \forall s \in \overline{\mathbb{S}} \tag{47}$$

Wages:

$$w_s = \frac{p_s A_s}{(1 + r_s^F)}, \quad \forall s \in \overline{\mathbb{S}} \tag{48}$$

Firm credit rate:

$$(1 + r_s^F) = \frac{B_s^{F\beta}}{L_s^{\beta F}}, \quad \forall s \in \overline{\mathbb{S}} \tag{49}$$

Prices

Interbank market rate:

$$(1+\rho) = \frac{B^{\beta\alpha}}{L^{\alpha\beta} + M^{CB}} \tag{50}$$

Commercial bank deposit rate:

$$(1+r^D) = \frac{(1+\rho)}{\left(1+\frac{1}{\epsilon^D}\right)} \frac{\mathbb{E}\left[R_s^\beta\right]}{\mathbb{E}\left[\nu_s^\alpha\right]} - \frac{\mathbb{E}\left[2\phi_s^\alpha(1-\nu_s^\alpha)^2 D^\alpha\right]}{\mathbb{E}\left[\nu_s^\alpha\right]} \tag{51}$$

Let $\vec{\sigma}^j$ denote the consumption, investment, and financing plans, i.e. vectors of decision variables, for each agent $j \in \{HH, F, \alpha, \beta, CB\}$.

$\vec{\sigma}^{HH} = (C_s^H, N_s^H, D^H, Z^H) \in \mathrm{R}^{S+1} \times \mathrm{R}^{S+1} \times \mathrm{R}^{S+1} \times \mathrm{R} \times \mathrm{R}$.

$\vec{\sigma}^F = (N_s^F, B_s^{F\beta}, \nu_s^F) \in \mathrm{R}^{S+1} \times \mathrm{R}^{S+1} \times \mathrm{R}^{S+1}$.

$\vec{\sigma}^\alpha = (D^\alpha, L^{\alpha\beta}) \in \mathrm{R} \times \mathrm{R}$.

$\vec{\sigma}^\beta = (L_s^{\beta F}, B^{\beta\alpha}, M_{F,s}, \nu_s^\beta) \in \mathrm{R}^{S+1} \times \mathrm{R}^{S+1} \times \mathrm{R} \times \mathrm{R}^{S+1} \times \mathrm{R}^{S+1}$.

Let $\vec{\eta} = \{p_s, W_s, r_s^F, \rho, r^D, r^Z\}$ denote the set of prices in the macroeconomy which are determined in equilibrium and which the household, firm, and banks take as given in their decision problem.

Let agent j's budget sets be given by $B^j(\vec{\eta})$.

Definition: GEI with banks and default

The set of allocations and prices $(\vec{\sigma}^j, \vec{\eta})$ is a monetary general equilibrium with incomplete markets (GEI), banks, and default if and only if:

1. 1.1 $\vec{\sigma}^{HH} \in \text{Argmax}_{\vec{\sigma}^{HH} \in B^{HH}(\vec{\eta})} \Pi^H$
   1.2 $\vec{\sigma}^{F} \in \text{Argmax}_{\vec{\sigma}^{F} \in B^{F}(\vec{\eta})} \Pi^F$
   1.3 $\vec{\sigma}^{i} \in \text{Argmax}_{\vec{\sigma}^{i} \in B^{i}(\vec{\eta})} \Pi^i$ for $i \in \{\alpha, \beta\}$.
2. All markets clear.

Equilibrium is characterised by rational expectations and market clearing.

Mechanism of the Model

Mechanism Step 1

Suppose that the central bank introduces CBDCs and raise interest rates on CBDCs.

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Through the household's Euler equations, they will only choose to hold bank deposits if the rate r^D on bank deposits is weakly greater than the rate on CBDCs:

$$\mathbb{E}\left[\beta^H \frac{C_0^H p_0}{C_s^H p_s} R_s^\alpha\right] = \frac{1}{(1 + r^D)} \qquad \text{(Bank Deposits)}$$

$$\mathbb{E}\left[\beta^H \frac{C_0^H p_0}{C_s^H p_s}\right] = \frac{1}{(1 + r^Z)} \qquad \text{(CBDCs)}$$

Mechanism Step 2

Deposit bank α raises deposit rates r^D to compete with CBDCs away from the monopsonistic level.

A higher deposit rate leads to an increase in deposits D^α.

Through deposit bank α balance sheet constraint, deposit bank α lends the additional deposits through the interbank market to bank β:

$$L^{\alpha\beta} = D^\alpha$$

Mechanism Step 3

By the equilibrium clearing condition for the interbank rate ρ, an increase in the supply of interbank loans leads to a decrease in the interbank rate:

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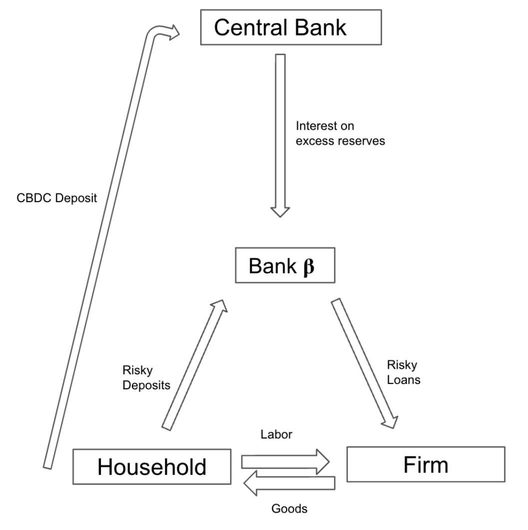
Bank β extends more risky loans to firms $L_s^{\beta F}$ **and** exerts less effort into monitoring these loans.

In the event of a bad shock, firms are more leveraged and have a higher marginal benefit to defaulting, thereby causing **financial contagion**.

DSGE

Model



Representative bank that deposits at the central bank.

Representative Household Problem

$$\max_{\{C_t, N_t, D_t, Z_t\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_H^t \left\{ log[c_t] - \chi_N \frac{N_t^{1+\psi}}{1+\psi} \right\} \right]$$

subject to:

$$C_t + D_t + Z_t \leq (1 + r_{t-1}^D) R_t^D D_{t-1} + (1 + r_t^Z) Z_{t-1} + W_t N_t + \Pi_t^F + \Pi_t^B$$

where $R_t^D \in [0, 1]$ is the expected repayment by the bank on the deposit and Π_t^F, Π_t^B are profit from firms and banks.

Representative Household Optimality Conditions

Let λ_t^H be the Lagrange multiplier on the budget constraint.

Household optimality conditions are:

$$\chi_N N_t^{\Psi} C_t = W_t \qquad \text{(Labor-Consumption)}$$

$$\mathbb{E}_t\left[\mathcal{M}_{t,t+1}^H (1 + r_t^Z)\right] = 1 \qquad \text{(CBDC)}$$

$$\mathbb{E}_t\left[\mathcal{M}_{t,t+1}^H (1 + r_t^D) R_{t+1}^D\right] = 1 \qquad \text{(Deposit)}$$

where $\mathcal{M}_{t,t+1}^H \equiv \beta_H \frac{C_t}{C_{t+1}}$ is the **stochastic discount factor**.

Representative Firm Problem

Firms face a cash-in-advance constraint to pay worker's wages.

$$\max_{\{N_t^F, B_t^{F\beta}, \nu_t^F\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta_F^t \left\{ \frac{(\Pi_t^F)^{1-\gamma^F}}{1-\gamma^F} - \phi_t^F [\mathbb{I}_t^F]^+ \right\} \right]$$

subject to:

$$W_t N_t^F \leq \frac{B_t^{F\beta}}{1+r_t^F}$$

$$Y_t^F = A_t N_t^F$$

$$ln(A_t) = \rho_A ln(A_t) + \epsilon_t^A$$

$$\Pi_t^F = Y_t^F - \nu_t^F B_{t-1}^{F\beta}$$

$$[\mathbb{I}_t^F]^+ = \left((1-\nu_t^F) B_{t-1}^{F\beta} \right)^2 \quad \text{if } \nu_t^F \in [0,1)$$

Representative Firm Optimality Conditions

Let λ_t^F be the Lagrange multiplier on the firm's cash-in-advance constraint.

Optimality conditions for the firm are:

$$\left(\Pi_t^F\right)^{-\gamma^F} = 2\phi_t^F(1-\nu_t^F)B_{t-1}^{F\beta} \qquad \text{(Optimal Default Condition)}$$

$$\frac{A_t}{W_t(1+r_t^F)}\left(\Pi_t^F\right)^{-\gamma^F} = \mathbb{E}_t\left[\beta_F 2\phi_{t+1}^F(1-\nu_{t+1}^F)B_t^{F\beta}\right] \qquad \text{(Optimal Borrowing Condition)}$$

Representative Bank Problem

Bank α and bank β collapsed into one bank.

Representative bank can hold reserves L_t^{CB} directly at the central bank.

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$$\max_{\{L_t^{CB}, L_t^F, M_t^F, \nu_t^\beta, D_t^H\}_{t=0}^\infty} \quad \mathbb{E}_0\Bigg[\sum_{t=0}^{\infty}\beta_\beta^t\Big\{\Pi_t^\beta - \frac{\gamma^\beta}{2}(M_t^F)^2 L_{t-1}^F - \phi_t^\beta[\mathbb{I}_t^\beta]^+\Big\}\Bigg]$$

subject to:

$$L_t^F + L_t^{CB} \leq D_t^H$$

$$M_t^F = \phi_t^F$$

$$\left(\Pi_t^F\right)^{-\gamma^F} = 2\phi_t^F(1-\nu_t^F)B_{t-1}^{F\beta}$$

$$\Pi_t^\beta = R_t^F(1+r_t^F)L_t^F + (1+r_t^{CB})L_t^{CB} - \nu_t^\beta(1+r_t^D)D_t^H$$

$$[\mathbb{I}_t^\beta]^+ = \left((1-\nu_t^\beta)D_t^H\right)^2 \quad \text{if } \nu_t^\beta \in [0,1)$$

Bank Optimality Conditions

Optimal deposit default condition:

$$(1 + r_t^D) = 2\phi_t^{\beta}(1 - \nu_t^{\beta})D_t^H \tag{52}$$

Optimal household deposit rate:

$$(1 + r_t^D) = \frac{(1 + r_t^{CB})}{(1 + \frac{\nu_t^{\beta}}{\epsilon_t^D})} \tag{53}$$

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Introducing CBDC grants the central bank greater control over the monetary policy transmission mechanism and induces perfect competition in the banking sector.

A higher bank deposit rate **crowds in** deposits into the banking sector.

By the **bank lending channel**, banks extend more credit in the interbank and firm credit market.

However, a lower interbank market rate reduces the incentive to monitor the credit worthiness of loans via the **risk-taking channel**.

More risky loans outstanding with banks having less equity buffer to withstand negative shocks, thereby generating **financial contagion**.

Tradeoff between monetary policy control and financial instability.

Future Work

Future Potential Work

- DSGE.

- Quantitative results of model.

- Relax assumption of perfect substitutability of CBDCs and bank deposits. Furthermore, allow CBDCs to also serve as transaction technology.